PART 1: QUESTIONS

Name:________________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Algebra II - Exam 1 **Lesson: 1-3**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. $F : A \to B$ is a function from set A (Domain) to set B (Codomain) that:

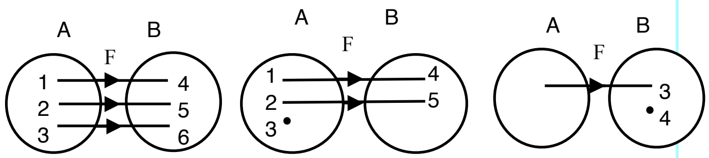
a) UNIQUE $y \in B \Rightarrow$ EVERY $x \in A$.
b) UNIQUE $x \in A \Rightarrow$ EVERY $y \in B$.
c) EVERY $y \in B \Rightarrow$ UNIQUE $x \in A$.
d) EVERY $x \in A \Rightarrow$ UNIQUE $y \in B$.
e) None of the above.

Solution: d

Definition of the function $F : A \to B$.

2. Which Graph(s) represent a function $F : A \to B$.

I. II. III.



Solution: b

a) Only I and II
b) Only I and III
c) Only II and III
d) I, II, and III are not functions
e) I, II, and III are functions.

Solution: b

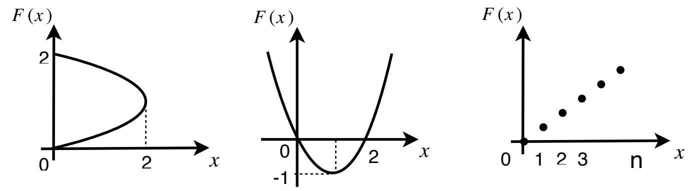
$F : A \to B$ is a function from set A (Domain) to set B (Codomain) that EVERY $x \in A \Rightarrow$ UNIQUE $y \in B$.

Note: II is False because it is not EVERY $x \in A$.

3. Which Graph(s) represent a function F on x.

I. II. III.

$F(x) : \mathbb{R} \to [0,2]$ $F(x) : [0,2] \to \mathbb{R}$ $F(x) : [0,2] \to \mathbb{R}$



a) Only II and III
b) Only I and III
c) Only I and II
d) Only III
e) I, II, and III are correct

Solution: c

$F : A \to B$ is a function from set A (Domain) to set B (Codomain) that EVERY $x \in A \Rightarrow$ UNIQUE $y \in B$.

Note: I is False because it is not a UNIQUE $y \in B$.
III is False because it is not EVERY $x \in A$.

4. Find the domain of the function $y = \dfrac{x}{x-5}$.

a) $D = \{x \in \mathbb{R} / x \neq 3\}$
b) $D = \{x \in \mathbb{R} / x \neq 4\}$
c) $D = \{x \in \mathbb{R} / x \neq 5\}$
d) $D = \{x \in \mathbb{R} / x \neq 6\}$
e) None of the above.

Solution: c

The denominator must be different from zero then $x - 5 \neq 0 \Rightarrow x \neq 5$.
Thus, $D = \{x \in \mathbb{R} / x \neq 5\}$.

5. Find the domain of the real function $y = x^2$.

a) $D = \{0\}$
b) $D = \{-5,6\}$
c) $D = \mathbb{R}$
d) $D = \{-5,0,2,3,6\}$
e) None of the above.

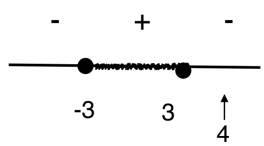
Solution: c

The parabola $y = x^2$ accepts $\forall x \in \mathbb{R}$. Thus, $D = \mathbb{R}$.

6. Find the domain of the real function $y : D \to \mathbb{R}$ such that $y = \sqrt{9 - x^2}$.

a) $D = \{x \in \mathbb{R} \,/\, x \leq -3 \text{ or } x \geq 3\}$
b) $D = \{x \in \mathbb{R} \,/\, x \leq -4 \text{ or } x \geq 4\}$
c) $D = \{x \in \mathbb{R} \,/\, -3 \leq x \leq 3\}$
d) $D = \{x \in \mathbb{R} \,/\, -4 \leq x \leq 4\}$
e) None of the above.

Solution: c



$y = \sqrt{9 - x^2}$

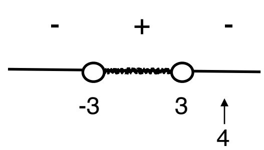
$9 - x^2 \geq 0 \Rightarrow (3 - x)(3 + x) \geq 0$

Thus, $D = \{x \in \mathbb{R} \,/\, -3 \leq x \leq 3\}$.

7. Find the domain of the real function $y : D \to \mathbb{R}$ such that $y = \dfrac{1}{\sqrt{9 - x^2}}$.

a) $D = \{x \in \mathbb{R} \,/\, x < -3 \text{ or } x > 3\}$
b) $D = \{x \in \mathbb{R} \,/\, x < -4 \text{ or } x > 4\}$
c) $D = \{x \in \mathbb{R} \,/\, -3 < x < 3\}$
d) $D = \{x \in \mathbb{R} \,/\, -4 < x < 4\}$
e) None of the above.

Solution: c



$y = \dfrac{1}{\sqrt{9 - x^2}}$

$9 - x^2 > 0 \Rightarrow (3 - x)(3 + x) > 0$

Thus, $D = \{x \in \mathbb{R} \,/\, -3 < x < 3\}$.

8. Let $F : A \to B$ be a function such that:

F is even $\Rightarrow F(-x) = F(x)$ for $\forall x \in A$.
F is odd $\Rightarrow F(-x) = -F(x)$ for $\forall x \in A$.

I. $F(x) = -3x^2 + 7$ is even.

II. $F(x) = \sqrt{(x)^2}$ is even.

III. $F(x) = |x|$ is even.

a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

Solution: d

I. True. F is even.

$F(x) = -3x^2 + 7 \Rightarrow$
$F(-x) = -3(-x)^2 + 7 = -3x^2 + 7 = F(x)$.

II. True. F is even.
$F(x) = \sqrt{(x)^2} \Rightarrow$
$F(-x) = \sqrt{(-x)^2} = \sqrt{(x)^2} = F(x)$.

III. True. F is even.
$F(x) = |x| \Rightarrow F(-x) = |(-x)| = |x| = F(x)$.

Thus, I, II, and III are correct

9. Let $F : \mathbb{R} \to \mathbb{R}$ be a function such that:

$$f(x) = \begin{cases} 1 & \text{for } x \leq -1 \\ 0 & \text{for } -1 < x < 1 \\ -1 & \text{for } x \geq 1 \end{cases}$$

Given $e = 2.71$ and $\pi = 3.14$. Calculate:

$$\frac{f(e) + f(-e)}{f(\pi) + f(-\pi)} = ?$$

a) Undefined.
b) 0
c) 1
d) 2
e) 3

Solution: a

$\dfrac{f(e)+f(-e)}{f(\pi)+f(-\pi)} = \dfrac{-1+1}{-1+1} = \dfrac{0}{0}$ (Undefined).

10. $F : A \to B$ is an injective function (one-to-one) if:
(Notation: "Im" is the image of F).

a) $x_1 \neq x_2 \Rightarrow F(x_1) \neq F(x_2); \forall x_1, x_2 \in A$.
b) $\exists x_1, x_2 \in A$ such that $F(x_1) = F(x_2)$.
c) $Im = B$.
d) $Im \neq B$.
e) None of the above.

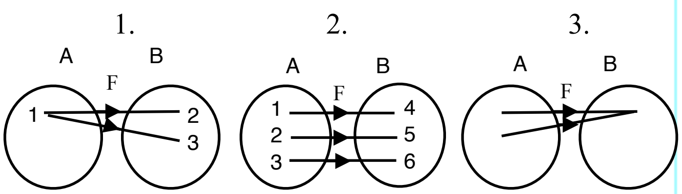
Solution: a

$F : A \to B$ is an injective function (one-to-one) if:

$x_1 \neq x_2 \Rightarrow F(x_1) \neq F(x_2); \forall x_1, x_2 \in A$.

11. Let $F : A \to B$ be a relation between A and B.

Assume: I) Injective function S) Surjective function
B) Bijective function N) Not a function.



Then:
a) 1-N, 2-S, and 3-B
b) 1-N, 2-B, and 3-S
c) 1-I, 2-S, and 3-N
d) 1-N, 2-S, and 3-B
e) None of the above.

Solution: b

$F : A \to B$ is a function from set A (Domain) to set B (Codomain) that EVERY $x \in A \Rightarrow$ UNIQUE $y \in B$.

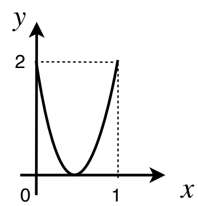
1-N: $F : A \to B$ is not a function because $\exists\, x \in A$ that doesn't correspond to a UNIQUE $y \in B$.

2-B: $F : A \to B$ is a bijective function because:
F is injective:
$x_1 \neq x_2 \Rightarrow F(x_1) \neq F(x_2); \forall x_1, x_2 \in A$

F is surjective: $\text{Im} = CD = B$. (Notation: "CD" is the Codomain of F).

3-S: $F : A \to B$ is a Surjective function because $Im = B$ (codomain).

12. Given the graph $y : [0,1] \to \mathbb{R}$:



a) y is surjective.
b) y is injective.
c) y is bijective.
d) y is not a function.
e) None of the above.

Solution: e

Note: $y : [0,1] \to \mathbb{R}$ is a function from set $[0,1]$ (Domain) to set $\mathbb{R}$ (Codomain) that:

EVERY $x \in [0,1] \Rightarrow$ UNIQUE $y \in \mathbb{R}$.

$y : [0,1] \to \mathbb{R}$ is not an injective function (one-to-one) because
$\exists x_1, x_2 \in [0,1]$ such that $F(x_1) = F(x_2)$.

$y : [0,1] \to \mathbb{R}$ is NOT a surjective function (onto) because $Im = [0,2] \neq CD = \mathbb{R}$. (Notation: "CD" is the Codomain of F).

13. Let $y : \mathbb{R} \to \mathbb{R}$ be a function such that $y = -2x + 1$. The inverse function y^{-1} is:

a) $y^{-1} = \dfrac{x-6}{2}$
b) $y^{-1} = \dfrac{-x+1}{2}$
c) $y^{-1} = \dfrac{-x+2}{3}$
d) $y^{-1} = \dfrac{x+3}{5}$
e) None of the above

Solution: b

$y = -2x + 1 \quad (x \leftrightarrow y)$

$x = -2y + 1$
$2y = -x + 1$
$y = \frac{-x+1}{2} \Rightarrow y^{-1} = \frac{-x+1}{2}$

14. Let $f : \mathbb{R} - \{0\} \to B$ such that:

$$f(x) = \frac{x+3}{3x}$$

The Im_f is:
Hint: $Im_f = D_{f^{-1}}$ (Image of f is the domain of its inverse f^{-1}).

a) $Im_f = \{y \in \mathbb{R} / y \neq 1\}$
b) $Im_f = \{y \in \mathbb{R} / y \neq \frac{1}{2}\}$
c) $Im_f = \{y \in \mathbb{R} / y \neq \frac{1}{3}\}$
d) $Im_f = \{y \in \mathbb{R} / y \neq \frac{1}{4}\}$
e) None of the above.

Solution: c

$y = \frac{x+3}{3x}$ $(x \leftrightarrow y)$
$x = \frac{y+3}{3y}$
$3xy - y = 3$
$(3x-1)y = 3$
$y = \frac{3}{3x-1} \Rightarrow f^{-1}(x) = \frac{3}{3x-1}$
$D_{f^{-1}} = \{x \in \mathbb{R} / x \neq \frac{1}{3}\}$.
Since $Im_f = D_{f^{-1}}$ then:
$Im_f = \{y \in \mathbb{R} / y \neq \frac{1}{3}\}$.

15. Given:

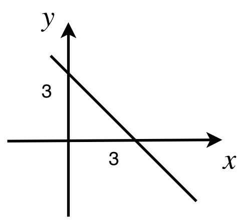
I. $y = 3 - \sqrt{x}$
II. $y = |x|$
III. $y = 2$.

a) Only I is a linear function.
b) Only II is a linear function.
c) Only III is a linear function.
d) Only II and III are a linear function.
e) None of the above.

Solution: e

Linear functions are in the form $y = mx + b$; where $m,b \in \mathbb{R}$ and $m \neq 0$.
Thus, there is no linear function.

16. Given the graph $y : \mathbb{R} \to \mathbb{R}$.



Then, the slope of y is:

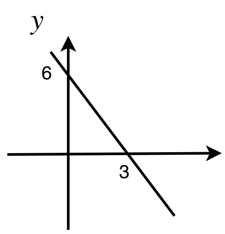
a) $m = -\frac{3}{2}$
b) $m = \frac{3}{4}$
c) $m = 3$
d) $m = -1$
e) None of the above.

Solution: d

Given $A(0,3)$ and $B(3,0)$.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow m = \frac{3-0}{0-3} = -\frac{3}{3} = -1.$$

17. The function of the graph $y : \mathbb{R} \to \mathbb{R}$ is:



a) $y = -2x + 6$
b) $y = -x + 4$
c) $y = 3x$
d) $y = 3x - 6$
e) None of the above.

Solution: a

Given $A(0,6)$ and $B(3,0)$.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow m = \frac{0-6}{3-0} = -\frac{-6}{3} = -2.$$

$y - y_0 = m(x - x_0) \Rightarrow y - 6 = -2(x - 0) \Rightarrow$
$y = -2x + 6.$

18. Given the straight line $(r)\ y = 3x + 2$. Find a straight line (s) that is parallel to the straight line (r) and passes through the point $A(0,1)$.

a) $y = 2x$
b) $y = -3x + 4$
c) $y = 3x + 1$
d) $y = -x$
e) None of the above.

Solution: c

$(r)\ y = 3x + 2 \Rightarrow m_r = 3$
Since $r // s$ then $m_r = m_s = 3$
$A(0,1)$ and $m_s = 3$
$y - y_0 = m(x - x_0) \Rightarrow y - 1 = 3(x - 0) \Rightarrow$
$(s)\ y = 3x + 1.$

19. Given:

$(r)\ y = \alpha^2 x + \pi$, where $\pi = 3.14$ and $e = 2.71$.

$(s)\ y = x - e$

Find α such that (r) is parallel to (s).

a) $\alpha = -1$ or $\alpha = 2$
b) $\alpha = 0$ or $\alpha = 1$
c) $\alpha = -2$ or $\alpha = 1$
d) $\alpha = 0$ or $\alpha = -1$
e) None of the above.

Solution: e

$(r)\ y = (\alpha^2)x + e \Rightarrow m_r = \alpha^2$
$(s)\ y = x - e \Rightarrow m_s = 1$

Since $r // s$ then $m_r = m_s \Rightarrow \alpha^2 = 1$

$\alpha^2 - 1 = 0 \Rightarrow (\alpha + 1)(\alpha - 1) = 0$

Thus, $\alpha = -1$ or $\alpha = 1$.

20. Given:

$(r)\ y = (\beta + 1)x + \sqrt[3]{2}$, where $\pi = 3.14$ and $e = 2.71$.

$(s)\ y = -2x - \sqrt[3]{2}$

Find β such that (r) is perpendicular to (s).

a) $\beta = 1$
b) $\beta = -1$
c) $\beta = 4$
d) $\beta = -\frac{1}{2}$
e) None of the above.

Solution: d

$(r)\ y = (\beta + 1)x + \sqrt[3]{2} \Rightarrow m_r = \beta + 1$

$(s)\ y = -2x - \sqrt[3]{2} \Rightarrow m_s = -2$

Since $r \perp s$ then $m_r . m_s = -1 \Rightarrow (\beta + 1)(-2) = -1$

$-2\beta - 2 = -1 \Rightarrow -2\beta = 1$

Thus, $\beta = -\frac{1}{2}$.

PART 2: SOLUTIONS

Consulting

Name:______________________________ **Age:**_____ **Id:**______________ **Course:**__________

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Calculate the domain of y:

$$y = \frac{x+2}{\sqrt[3]{x}}$$

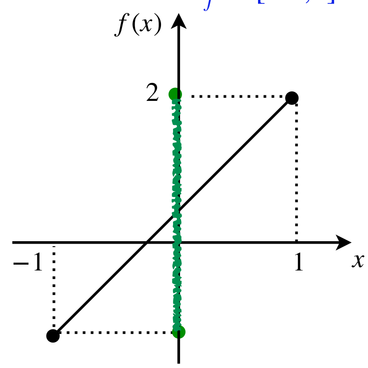
Solution: $D = \{x \in \mathbb{R} \,/\, x \neq 0\}$.

$\sqrt[3]{x} \neq 0 \Rightarrow x \neq 0$.
Thus, $D = \{x \in \mathbb{R} \,/\, x \neq 0\}$.

22. Let $f : [-1,1] \rightarrow \mathbb{R}$ such that:

$f(x) = \frac{3}{2}x + \frac{1}{2}$. Find the image of f. (Hint: Draw a graph)

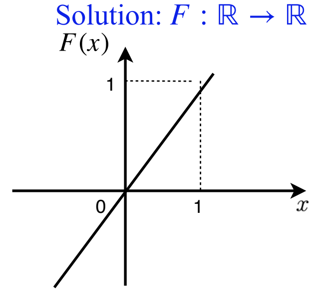
Solution: $Im_f = [-1,2]$.



Thus, $Im_f = [-1,2]$.

23. Give one example of a bijective function.

Solution: $F : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x$.

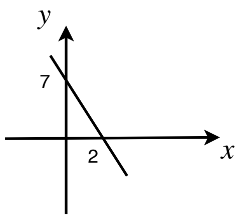


F is injective: $x_1 \neq x_2 \Rightarrow F(x_1) \neq F(x_2); \forall x_1, x_2 \in \mathbb{R}$.
F is surjective: Im $= CD = \mathbb{R}$.

24. Find the graph of the linear function:

$$\frac{x}{2} + \frac{y}{7} = 1$$

Solution:



25. Given:

$$f(x) = \frac{x+1}{x-2}$$

Find the domain (D_f) and the image (Im_f).

Solution: $D_f = \{x \in \mathbb{R} \,/\, x \neq 2\}$ and $Im_f = \{y \in \mathbb{R} \,/\, y \neq 1\}$.

Domain:

$x - 2 \neq 0 \Rightarrow x \neq 2$

Thus, $D_f = \{x \in \mathbb{R} \,/\, x \neq 2\}$.

Image:

$y = \dfrac{x+1}{x-2}$ $(x \leftrightarrow y)$

$x = \dfrac{y+1}{y-2}$

$xy - 2x = y + 1$

$xy - y = 2x + 1$

$y(x-1) = 2x + 1$

$y = \dfrac{2x+1}{x-1}$

$x - 1 \neq 0$

$D_{f^{-1}} = \{x \in \mathbb{R} \,/\, x \neq 1\}$. Since $Im_f = D_{f^{-1}}$

Thus, $Im_f = \{y \in \mathbb{R} \,/\, y \neq 1\}$.